# Makespan minimization in tree data gathering networks with memory limits

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## 1 Introduction

A data gathering network is a computer system comprising a set of worker nodes and a base station. The workers obtain or produce datasets which have to be transferred to the base station. Data gathering wireless sensor networks find applications in military surveillance, environment monitoring and healthcare. A distributed system, where the computation results obtained by many workers have to be passed to a single server, is also a data gathering network.

In a star data gathering network, the workers communicate directly with the base station. Contrarily, in a tree network, data are relayed through a set of intermediate nodes. Scheduling algorithms were designed for optimizing data gathering in several types of star networks, such as, e g., networks with data compression [1, 5, 6], with variable communication speed [3] and with limited base station memory [2]. Tree data gathering networks have attracted less attention so far. Minimizing the maximum lateness in a tree network with dataset release times and due dates was studied in [4].

This work considers scheduling in a tree data gathering network with limited memory. Each of the worker nodes holds a dataset that has to be sent to an appropriate intermediate node. The intermediate node processes the dataset and then sends it to the base station. A dataset occupies the intermediate node's memory buffer from the moment when it starts being received until the time when its transfer to the base station completes. The total size of datasets coexisting in the memory of an intermediate node can never exceed its buffer size. Our goal is to organize dataset transfers and processing so as to minimize the total time required to gather the data.

## 2 Problem formulation

The data gathering network consists of n worker nodes, m intermediate nodes and a single base station. An intermediate node  $P_i$ , where  $1 \le i \le m$ , gathers data

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from  $n_i$  workers  $P_{ij}$ , where  $1 \leq j \leq n_i$ . Thus,  $n_1 + \cdots + n_m = n$ . A worker  $P_{ij}$ holds a dataset  $D_{ij}$  of size  $\alpha_{ij}$ , which has to be sent to the intermediate node  $P_i$ for processing. The processed dataset is then passed by  $P_i$  to the base station. Sending dataset  $D_{ij}$  from  $P_{ij}$  to  $P_i$  takes time  $c_1 \alpha_{ij}$ , and processing this dataset by  $P_i$  requires time  $a\alpha_{ij}$ . The processed dataset  $D_{ij}$  is sent to the base station in time  $c_2 \alpha_{ij}$ . Preemptions in communication or computation are not allowed. Each node, including the base station, can receive at most one dataset at a time. An intermediate node can process at most one dataset at a time, but it can receive one dataset, process another dataset, and send yet another dataset simultaneously. Each intermediate node processes and sends datasets in the order in which it receives them. The size of the memory buffer of node  $P_i$  is  $B_i \geq \max_{i=1}^{n_i} \alpha_{ij}$ . At the moment when a dataset  $D_{ij}$  starts being sent to  $P_i$ , a memory block of size  $\alpha_{ij}$ is allocated at  $P_i$ . The block is released when the transfer of the processed dataset  $D_{ij}$  to the base station is completed. The total size of memory blocks allocated at  $P_i$  cannot at any time exceed  $B_i$ . The scheduling problem is to minimize the makespan, i.e., the time by which all processed datasets arrive at the base station.

#### 3 Results

The analyzed problem generalizes makespan minimization in a star data gathering network with limited base station memory [2], and hence, it is strongly NP-hard. We first formulate it as an integer linear program (ILP). The program contains O(n) rational variables,  $O(n^3)$  binary variables and  $O(n^3)$  constraints, and in consequence, it cannot be used in practice for solving even moderate size instances. Therefore, we also propose heuristic algorithms.

First, we design simple heuristics running in  $O(n^2)$  time. Each of these algorithms is defined by two rules. The first one determines the order in which the datasets are transferred to each intermediate node  $P_i$ , and the second one defines the order in which the datasets are passed to the base station. We consider the following rules for sending datasets to intermediate nodes.

- Inc: send the datasets in the order of non-decreasing sizes  $\alpha_{ij}$ .
- LF: always choose the largest dataset which fits in the memory currently available at the intermediate node.
- Rnd: send the datasets in a random order.

Since the datasets are sent to the base station by an intermediate node  $P_i$  in the order in which they were received and processed, a rule designed for sending the datasets to the base station only has to choose the order of the datasets received from different intermediate nodes. We analyze two scheduling methods for this stage.

• FIFO: transfer the datasets in the order in which their processing at the intermediate nodes completed.

• B: choose the dataset from the intermediate node which has the smallest currently available memory.

An algorithm that uses Rule1 for sending the datasets to the intermediate nodes, and Rule2 for transferring them to the base station, is denoted by Rule1-Rule2.

Furthermore, we design a variable neighborhood search algorithm VNS. In this algorithm, a schedule is represented by an array x[1..n] of dataset priorities, where a smaller number means a higher priority. The priority of dataset  $D_{ij}$  is  $x[\sum_{k=1}^{i-1} n_k + j]$ . In order to compute the makespan, the schedule is constructed using the defined priorities both for sending the datasets to the intermediate nodes, and for transferring them to the base station. Variable neighborhood search consists in systematically changing the neighborhoods used during local search. Suppose a sequence of neighborhoods  $N_1, \ldots, N_{k_{max}}$  are defined. The algorithm starts with the current neighborhood number k = 1 and a given initial solution x. In each step, the best solution x' in neighborhood  $N_k(x)$  is found. If x' is better than x, then x is changed to x' and k is set to 1. If x' is not better than x, then k is increased by 1. The search continues until k exceeds  $k_{max}$ . In our algorithm, the initial solution is  $x = [1, \ldots, n]$ , and the following three neighborhoods are used.

- $N_1(x)$  contains all arrays obtained from x by reversing any subarray x[i..j].
- $N_2(x)$  contains all arrays obtained from x by swapping a pair of values x[i] and x[j].
- $N_3(x)$  contains all arrays obtained from x by moving a value x[i] to an arbitrary position  $j \neq i$  in the array.

The proposed algorithms were implemented in C++ and tested in a series of computational experiments on randomly generated instances. The obtained results lead us to the following conclusions.

- The computational cost of ILP is very high even for small instances.
- The B rule for scheduling the transfers to the base station is counterproductive. Much better results are obtained by the FIFO rule.
- Among the simple heuristics, the best solutions are usually produced by LF-FIFO. Still, if  $c_2$ ,  $\delta_B$  or m is very large, or  $c_1$  or a is very small, then Inc-FIFO obtains better schedules.
- VNS significantly outperforms the simple heuristics. It delivers very good results in a short time.

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